Part I (95 pts)

1. Find the length of the arc for $\mathbf{r}(t) =< \cos t\,, \sin t\,, \ln \cos t >, \quad 0 \le t \le \frac{\pi}{3}$.

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$$= \int_0^{\pi/3} \sqrt{\underbrace{\sin^2 t + \cos^2 t}_{=1} + \tan^2 t}\, dt$$

$$= \int_0^{\pi/3} \sqrt{\sec^2 t}\, dt = \int_0^{\pi/3} \sec t\, dt$$

$$\frac{\sec t + \tan t}{\sec t + \tan t}$$

$$= \ln|\sec t + \tan t| \Big]_0^{\pi/3} = \ln|2+\sqrt{3}| - \cancel{\ln|1+0|}\ 0$$

$$= \ln(2+\sqrt{3})$$

Let $\mathbf{r}(t) =< 2\cos t\,, 2\sin t\,, t^2 >$ for $t = 0$.

2. Find an equation of the normal plane. T.

$$T = \frac{r'}{|r'|} = \langle 0, 1, 0 \rangle \qquad r\Big|_{t=0} = \langle 2, 0, 0 \rangle$$

$$r' = \langle -2\sin t, 2\cos t, 2t \rangle \overset{t=0}{=\!=} \langle 0, 2, 0 \rangle$$

$$|r'| = \sqrt{4 + 4t^2} = 2\sqrt{1+t^2} = 2$$

$$0(x-2) + 1(y-0) + 0(z-0) = 0 \qquad \boxed{y = 0}$$

3. Find the unit tangent vector at $t = 0$.

$T = \langle 0, 1, 0 \rangle$ from #2.

4. Find the curvature at the point $t = 0$..

$$K = \frac{|T'|}{|r'|}$$

$$T = \left\langle \frac{-\sin t}{\sqrt{1+t^2}}, \frac{\cos t}{\sqrt{1+t^2}}, \frac{t}{\sqrt{1+t^2}} \right\rangle$$

$$T' = \left\langle \frac{-\cos t\sqrt{1+t^2} + \cancel{\sin t\left(\frac{2t}{2\sqrt{1+t^2}}\right)}}{1+\cancel{t^2}}, \frac{\cancel{-\sin t\sqrt{1+t^2}} - \cos t\cancel{\left(\frac{2t}{2\sqrt{1+t^2}}\right)}}{1+\cancel{t^2}}, \frac{\sqrt{1+t^2} - \cancel{t\left(\frac{2t}{2\sqrt{1+t^2}}\right)}}{1+\cancel{t^2}} \right\rangle$$

$$= \langle -1, 0, 1 \rangle$$

$|T'| = \sqrt{2}$

$|r'| = 2$

$$K = \frac{\sqrt{2}}{2}$$

Part II (15 pts)

5. Find the curvature of the curve with parametric equations.

$r \Rightarrow \quad x = \int_0^t \sin\left(\frac{1}{2}\pi\theta^2\right) d\theta \qquad y = \int_0^t \cos\left(\frac{1}{2}\pi\theta^2\right) d\theta$

$r' = \langle \sin \frac{1}{2}\pi t^2, \cos \frac{1}{2}\pi t^2 \rangle \qquad |r'| = 1$

$T = \frac{r'}{|r'|} = r'$

$T' = \langle \pi t \cos \frac{\pi}{2} t^2, -\pi t \sin \frac{\pi}{2} t^2 \rangle$

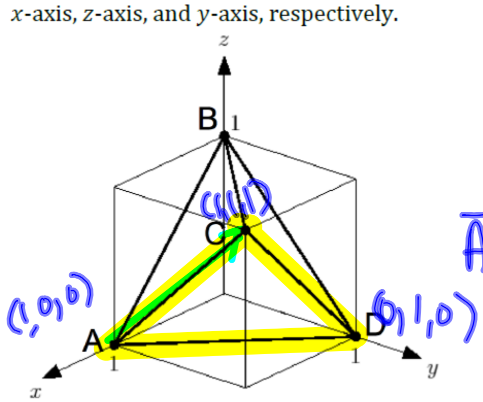
$|T'| = \sqrt{\pi^2 t^2 (1)}$

$= \pi t$

$K = \frac{\pi t}{1} = \pi t$

Part I (95 pts)

A unit cube is placed in the first octant, where C is at (1, 1, 1) and A, B, and D, are on x-axis, z-axis, and y-axis, respectively.



$\vec{AC} = \langle 0, 1, 1 \rangle$

$\vec{AD} = \langle -1, 1, 0 \rangle$

1. Evaluate $\vec{AC} \cdot \vec{AD}$.
2. What is the measure of angle CAD?

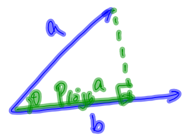
$\vec{AC} \cdot \vec{AD} = 0 + 1 + 0 = 1$

$\vec{AC} \cdot \vec{AD} = |\vec{AC}||\vec{AD}| \cos\theta$

$1 = \sqrt{2} \cdot \sqrt{2} \cos\theta$

$\frac{1}{2} = \cos\theta \qquad \theta = \frac{\pi}{3}$

3. Find the projection vector of AC on AD.
4. Find a vector that is perpendicular to both $\overrightarrow{AC}$ and $\overrightarrow{AD}$.



$\text{Proj}_b\, a = |a|\cos\theta =$

$\text{Proj}\ \overrightarrow{AC} = |\overrightarrow{AC}|\cos\theta$

$= \dfrac{\overrightarrow{AC}\cdot\overrightarrow{AD}}{|\overrightarrow{AD}|} = \dfrac{1}{\sqrt{2}}$

$\text{Proj}\ \overrightarrow{AC}\ \text{vector} = \dfrac{1}{\sqrt{2}}\dfrac{\overrightarrow{AD}}{|\overrightarrow{AD}|}$

$= \dfrac{\langle -1, 1, 0\rangle}{2}$

4) $\overrightarrow{AC} \times \overrightarrow{AD}$

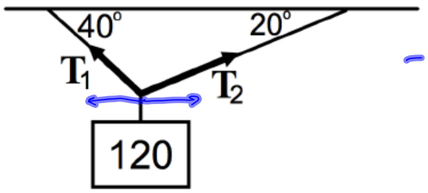
$= \langle 0, 1, 1\rangle \times \langle -1, 1, 0\rangle$

$= \langle -1, -1, 1\rangle$

5. Find the area of Triangle ACD.

$\frac{1}{2}\left|\overrightarrow{AC} \times \overrightarrow{AD}\right| = \frac{1}{2}\sqrt{3} = \frac{\sqrt{3}}{2}$

6. A 120-lb weight hangs from two wires as shown in the figure below. Find the tensions (forces) $\mathbf{T}_1$ and $\mathbf{T}_2$ in both wires and their magnitudes.



$$-T_1\cos 40^\circ + T_2\cos 20^\circ = 0$$

$$T_1 = \frac{T_2\cos 20}{\cos 40}$$

$$T_1\sin 40^\circ + T_2\sin 20^\circ - 120 = 0$$

$$T_2\tan 40\cos 20 + T_2\sin 20 - 120 = 0$$

$$T_2 = \frac{120}{\tan 40\cos 20 + \sin 20} = 106$$

$$T_1 = 130$$